How many angels can dance on the point of a needle?

1. Indefinite extensibility and possibilia

- *Kaplan's paradox for possible worlds accounts of propositions*:
  - *Plenitude*: $\exists p \forall w (w \models p \leftrightarrow ...w...)$

  Define $\Phi(p,w)$: "p is thought by thinker S at time t at w"
  - *Totality*: $\forall p \exists w \, \Phi(p,w)$
  - *Uniqueness*: $\forall w \forall p \forall p' (\Phi(p,w) \wedge \Phi(p',w) \rightarrow p = p')$

  We can deduce a contradiction from the following instance of Plenitude:

$$\exists p \forall w (w \models p \leftrightarrow \exists q (\Phi(q,w) \wedge \neg w \models q))$$

- *Forrest-Armstrong style objection to modal realism*:
  - *Recombination*: No matter what some objects are, there is a world w which copies them, i.e., contains distinct duplicates of all of them.
  - *No necessary connection between distinct existences*: Whenever distinct objects co-exist at a world, any of them can exist at a world without the others.

  Define $\Psi(p,w)$: w copies all and only possible worlds in which p is true.

Two lines of response:

- Deny some auxiliary hypothesis and maintain that no relation between propositions and possible worlds ever satisfies Totality and Uniqueness.
- Invoke indefinite extensibility and deny that quantification over possibilia is ever absolutely general.

2. The cardinal problem of absolute generality

2.1. How many angels can dance on the point of a needle?

> *How many angels can dance on the point of a very fine needle, without jostling one another?* D'Israeli (1875)

(i) Not only can angels inhabit places, they can inhabit point-sized regions.

(ii) No bar on angel cohabitation.

(iii) Different pluralities of angels have different fusions.

Some offensive answers:

- Exactly seven (or seventeen or ninety four or ...)
- There could not be infinitely many angels, but for every finite cardinal number, there could be exactly that many angels.
- Exactly aleph-seven (or aleph-seventeen or aleph-ninety four or ...)

2.2. Two live answers

Indefinite Extensibility: There could not be so many angels as to exceed each and every aleph, but for each aleph, there could be exactly that many angels.

Plenitude: There could be at least as many angels as there are alephs.

2.3. Two argumentative tools

- A plurality is *disperse* iff no one object fuses two different subpluralities of it.
- A fusion is *based on* a plurality iff it fuses one of its subpluralities.

(A) If a plurality is disperse and more numerous than one, then there will be more fusions based on that plurality than there are members of that plurality.

(B) Limitation of Size: Tthere is exactly one size beyond all the aleph-sizes.

2.4. Plenitude under fire

The Limitation of Size Argument:

Any angels have a fusion. By (A) above, there are strictly more fusions of angels than there are angels. But, by Plenitude and Limitation of size, the angels already have the *only* size available beyond all the aleph-sizes. So, the fusions of angels cannot be strictly more numerous than all the angels. Contradiction.

2.5. Trouble for modal realism

1. By Indefinite Extensibility, there are, across possible worlds, as many possible angels as there are alephs.
2. There is no bar on transworld composition.
3. By Limitation of Size, the possible angels across all possible worlds have the *only* size available beyond all the aleph-sizes. But by transworld composition and our mereological result, the fusions of angels across possible worlds must be strictly more numerous than the angels. Indefinite Extensibility has to give.

But, then, some aleph must set an upper bound on the size angels can have!

2.6. Trouble for necessitarianism

1. By Indefinite Extensibility, there are as many possible angels as there are alephs.
2. Every possible angel exists in the actual world, though perhaps not as an angel.
3. By Limitation of Size, the possible angels have the *only* size available beyond all the aleph-sizes. But by intra-world composition and our mereological result, the fusions of possible angels must be strictly more numerous than the possible angels. Indefinite Extensibility has to give again.

3. Some ways out

References

D'Israeli, I. (1875). Quodlibets, or scholastic disputations. In Griswold, R. W., editor, *Curiosities of Literature and the Literary Character Illustrated, with Curiosities of American Literature*. New York.