Semantics for Plurals

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Workshop on Absolute Generality
Paris, ENS, September 8, 2009

Introduction

First- and higher-order logic contain singular quantifiers, like the existential quantifier (something).
But many natural languages have plurals and collective predicates.

John and Bill are young = John is a young and Bill is young.
But *John and Bill met* doesn't mean that John met and Bill met.
How can the semantics of collective sentences be characterized?

* Singularism (most natural language semanticists):
Use first- (or higher-) order logic, together with sets or sums

* Pluralism (McKay, Oliver & Smiley, Rayo, Yi):
Use logics enriched with plural quantifiers

* For a semanticist like Link:

Some men carried a piano (together) is true iff
∃s [s is a sum of men & carried-piano(s)]
Something is a sum of men and it carried a piano.

* For plural logicians, this is unsatisfactory.
Plural quantification and predication are primitive, they should be added directly to the logic:

Some men carried a piano (together) is true iff
∃xs [men(xs) & carried-piano(xs)]
Some things are men and they carried a piano.

* <u>Why is the debate between singularism and pluralism important?</u>

First, ideally, semanticists would like to have the most adequate semantics for plurals.

Second, plural logic has become a very popular tool in the philosophy of mathematics and metaphysics because:
- Plural logic has the expressive and deductive power of monadic second-order logic.
- Unlike second-order logic, plural logic may seem ontologically innocent.

But of course, this isn't the case if singularism is right.

* In this talk, I want to review what the various positions and arguments are, indicating where we may be at a stand-off and where progress has to be made.

* Absolutely general quantification?
As we'll see, an important argument against singularism is based on the assumption that we can quantify over absolutely everything...

PLAN

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§1. What may we expect of a semantics of plurals?

* A specification of the truth-conditions of sentences containing plurals:

Starting point: paraphrases of various kinds.

Better still: a full-fledged truth-theory, ie a *systematic* translation in an interpreted language.

Ideal: compositionality (may be unattainable for natural language).

* A model theory to characterize the notion of logical consequence.

§2. Radical mereological singularism (and two problems)

Goodman & Quine (1947) and Goodman (1951), with roots in Leonard & Goodman (1940) and Lesniewski (1919)

Basic idea: the semantic values of plural expressions are (mereological) sums.

John and Mary surrounded the (Lilliputian) castle is true just in case the sum of John and Mary surrounded the castle.

A plural predication is thus reducible to a singular predication, in which a property is ascribed to one object, a certain sum.

What are (mereological) sums?

Many alternative axiomatizations are a priori possible (Simons 1987, Varzi 2009). But the most popular is *classical extensional mereology*.

a overlaps b just in case something is part of a and part of b.

The sum of everything that satisfies Q is the object $s = \sigma[x / Q(x)]$ such that something overlaps s just in case it overlaps something that satisfies Q.

Particular case: a+b is the mereological sum of a and b.

Axioms:

- Partial ordering + Strong supplementation

=> The mereology is extensional: objects that have the same parts are identical.

- Axiom schema guaranteeing that the sum of everything that satisfies Q exists whenever something satisfies Q.

First motivation: Multigrade predicates (Leonard & Goodman 1940)

Most predicates in English appear to take different numbers of arguments at the same argument place:

(2a) *John and Mary surrounded the castle*.

(2b) *John, Mary, and Bill surrounded the castle*.

One simple way to deal with this phenomenon is to use sums.

It's always a single object (a certain sum) that surrounds the (Lilliputian) castle.

Second motivation: Nominalism

(Goodman & Quine 1947, Goodman 1951)

'We do not believe in abstract entities: classes [ie sets], relations, properties, etc'. 'We renounce them altogether'.

Then they consider some of the nominalist's assets and problems.

Some talk about sets is easily reducible:

The set of cats is included in the set of mammals is true iff every cat is a mammal.

But what about mathematical discourse generally?

And what about some plural sentences?

There are more dogs than cats.

Let's look at two problems discussed by friends of plural logic.

First problem: Two men wrote a book, but the sum of their molecules did not

Yet, the mereological sum of the men is identical with the sum of their molecules (Oliver and Smiley 2001).

Intuitive answer: The sums are in fact distinct: the sum of the molecules survives the dispersion of the molecules, while the sum of the men does not since the men do not continue to exist.

We can say that a molecule is part of a man, and that a man is part of two men. But is it the same relation of part? If it is and the relation is extensional, we have a problem.

So we must give up extensionality.

One way to do so is to use at least two notions of part.

Link (1983, 1998):

- Domain of individuals: relation of individual part; the domain is closed for individual sums. (Mary is an atomic individual.)

- Domain of matter: relation of material part; the domain is closed for material sums.

- A function associates to each individual its matter.

We can then coherently deny that the (individual) sum of the men is identical to the (individual) sum of their molecules.

Second problem: Counting
Oliver & Smiley (2001), Yi (2005), McKay (2006)

The men are two is true iff the (individual) sum of the men is two.
But the (individual) sum of the men is one.

Answer: Follow Frege (1884) and Geach (1962), like Link (1998). Counting requires the identification of a sortal or count noun specifying what is to be counted.

The men are two is understood as *The men are two men.*
The (individual) sum of the men is one is understood as *The (individual) sum of the men is one (individual) sum of the men.*

Formally, and more generally:
The men are n (and not more) is true iff there exists a unique s such that:
i) $s = \sigma_i[x / man(x)]$
ii) $\exists x1 \ldots \exists xn [s = x1 +_i \ldots +_i xn \wedge \forall i (man(xi)) \wedge \forall i \neq j (xi \neq xj)]$
iii) An analog of condition ii) cannot be satisfied for a number l greater than n

Counter-argument: Suppose the universe contains just a and b ($a \neq b$), and their (individual) sum c. a is the (individual) sum of itself, and so is b. So intuitively, these two sentences are true:

a and b are two individual sums.
a, b and c are three individual sums.
But $a +_i b = a +_i b +_i c$, so the theory just outlined predicts that *a and b are three individual sums* should be true.

So Link's move of using individual sums works well, except when we proceed to count individual sums themselves.

Possible answer: The metalanguage contains additional logical and mereological notions. Those are not innocent since the semantics associates to any ordinary individuals a *different* object, their individual sum.

Therefore, the specialized vocabulary used in the metalanguage *cannot apply to itself.* When doing the semantics of the metalanguage, a different, meta-metalanguage must be used, with a new battery of mereological predicates. And so ad infinitum.

Why should such a hierarchy be more problematic than the hierarchies postulated, among others, by the friends of plural logic (Rayo 2006)?

Nonetheless, life is more difficult without sets, especially so if we want to do semantics and develop a model theory.

§3. Liberal mereological singularism
Link (1983, 1998), Gillon (1992)

Sums are still the referents of plural expressions.

But otherwise, set theory is used as it normally is in natural language semantics.

(So the basic ontology is enriched: the domain of discourse contains ordinary individuals plus all their (individual) sums.)

Motivations:

- Multigrade predicates

- Develop semantics as usual

- While sets are abstract entities, sums need not be: the sum of John and Mary is concrete. So while their set could not carry a piano, their sum could.

- Treat plurals and mass nouns (*wine*, *furniture*) in similar (though not identical) ways.

What about our earlier problems?

* The men wrote a book, but the sum of their molecules did not
This still requires to use a non-extensional mereology.
We can use Link's, with individual sums for plurals, and material sums for matter; but other options remain open as well.

* Counting problems:

Link's strategy is still available, of course, with its consequences (hierarchy of metalanguages).

Can we proceed differently, with the help of sets?

Gillon (1992) associates to any count noun phrase its denotation, which is a set:

[*the men*] = [*man*] = {a,b}, and [*the women*] = [*woman*] = {c,d}

Then [*the men and the women*] = [*the men*] $\cup$ [*the women*] = {a,b,c,d}

Sums come into play via the notion of 'aggregation'.

The men and the women carried a piano may mean that:
- each of them carried a piano (the distributive construal)
- they all together carried the piano (the collective construal)
- the men together carried a piano, and so did the women (an intermediate construal)

Gillon: The interpretation of the sentence depends on the choice of an aggregation over the subject's denotation. The sentence then asserts that the main predicate is satisfied by each member of the chosen aggregation:
- distributive construal: aggregation = {a,b,c,d}
- collective construal: aggregation = {a+b+c+d}
- intermediate construal: aggregation = {a+b,c+d}

So sums are the subjects of collective predications.

Then what about:
The men and the women carried a piano (together).
The men and the women are four.

Under Gillon's semantics, the first sentence is true because carried-piano(a+b+c+d).

What about the second sentence? Could Gillon say this?
Ascriptions of number, though collective, are lexically special:
They are calculated from the noun phrase's denotation.

How problematic is that?

§4. Set singularism
Landman (1989), Schwarzschild (1996), among others

The key difference with liberal mereological singularism =
The referent of a plural expression (what a collective predicate applies to) isn't a sum anymore, but a set.

Landman (1989) 'identifies' individuals to their singleton set.

If the real individuals are John and Mary, the semantics assigns to the constant *John* the singleton set {John}, and to *Mary* the set {Mary}.
The predication *John carried a piano* is true iff the singleton set {John} is a member of the denotation of *carried a piano*.

Then similarly, a collective predication of *John and Mary carried a piano* is true iff {John, Mary} is a member of the denotation of *carried a piano*.

Motivations:

- Convenience: why use both sets and sums, if we can use only sets?

- The historical connection between plurals and 'classes of things' (ie sets of things). So perhaps what the use of the plural does is to form a set (cf. Russell 1919).

How significant is the difference between set and mereological singularism?

- Pragmatically: according to Landman, in order to characterize the readings of plural sentences, using sets or sums makes no difference.

- Concerning concreteness: Sums of concrete things are concrete, while sets are often said to be abstract.

Well: is there anything in set theory itself that entails that sets of concrete things could not be concrete, have causal powers?

- Ontologically: It is sometimes said that a sum of men is identical to its parts (the men), while a set of men is distinct from its members.

However, composition as identity is problematic if taken literally; cf Sider (2007, submitted) for very weird consequences of the view.

§5. Second-order singularism
Bennett (1975), Hausser (1974), following Montague (cf. Lonning 1997)

Plurals would denote properties, while collective predicates would denote second-order properties.

The twelve students gathered is true iff
TWELVE(STUDENT) & GATHERED(STUDENT)
where STUDENT is a first-order property,
and TWELVE and GATHERED are second-order

Under the usual semantics for second-order logic, a first-order property is interpreted as a set, and a second-order property as a set of sets. So this becomes a form of set singularism.

Still, other semantics are conceivable (Williamson (2003) suggests one possibility).
Then a second-order (or higher-order) approach could be something genuinely different.
But under what form could it still count as singularist?

§6. Event singularism
Lasersohn (1995) and Schein (1993) use events for the semantics of plurals.

They do so in order to characterize better the readings to which plural sentences are liable.

But using events doesn't seem to make a real difference concerning the debates between singularism and pluralism (cf. Rayo 2002).

§7. Ontological commitment, or plural innocence?
Yi (2005), McKay (2006), Linnebo (2008, §5)

For the singularist:
John and Mary carried a piano (together) is true iff the sum / set of John and Mary carried a piano.
The right-hand side implies that there exists a sum / set.

But the original, English sentence would not.
And more generally, plural sentences (and hence, plural logic) would be ontologically innocent.

(NB: This alleged innocence is an important reason why plural logic seems so attractive in metaphysics.)

First answer: The purpose of such bi-conditionals is to characterize inferential relations between various sentences of English.
The metalanguage used brings its own commitments (to sums / sets), independent of those of sentences of English.

Response: Ideally, a semantic theory for natural language should characterize inferential relations in a transparent fashion, and in a way which makes clear what the ontological commitments of English sentences are.

Well, is that so? That's not completely obvious.

But more crucially:

Second answer: The ontological commitments of a sentence and the inferences that can be drawn from it are difficult to assess.
One should not rely only on hunches and intuitions.

Sums and sets are theoretical notions, and so it's indeed not clear whether an inference to their existence holds.

But there are similar inferences that seem quite natural:
The men surrounded the castle.
=> *A group of men surrounded the castle*.
And perhaps a group of men *is* a sum or set of men, this very fact failing to be transparent to ordinary speakers.

Parallel case: If Davidson is right, the semantics of many sentences involves existential quantification over events:
John walked is true iff ∃e (walking(e) & agent(e,John))
But the existence of this event isn't clear on the basis of intuitions about the meaning of *John walked*.
If Davidson's semantics is attractive, its because of its theoretical merits, not solely or primarily because of intuitions.

Also, on a simpler analysis, *John walked* is true iff walked(John).
Because of this, for Frege, the sentence is committed both to the object John and to the function walked(). And why not indeed?

Overall, here, I think we have a stand-off between competing and indecisive intuitions and arguments.

§8. Absolute generality versus indefinite extensibility
Cf. Linnebo (2008, §4.3)

In mathematics and its philosophy, we may want to talk about absolutely all sets:

Any set satisfies the axioms of set theory.

There are some sets such that a set is one of them just in case it is not a member of itself.
(i.e. *There are some sets which are all and only the non-self-membered sets*)

What semantics can we propose for the 'Russelian' sentence?

* First, using sets, we may propose:
$\exists y\ \forall x\ (x \in y \leftrightarrow x \notin x)$
But if the quantifiers ∃y and ∀x range over the same sets and if y exists, we get a contradiction:
$y \in y \leftrightarrow y \notin y$
So if the sentence does range over all sets, and it is translated as proposed, then the sentence must be false (y cannot exist), while it seems to be genuinely true.

One response is in terms of indefinite extensibility.
(Cf. Dummett 1981, Glanzberg 2004, 2006)
The translation proposed *is* adequate, but the range of the quantifier ∃y is bigger than that of ∀x.

So Russell's paradox shows that, contrary to first appearances, we *cannot* quantify over absolutely all sets.
The concept of set is indefinitely extensible: whenever we have formed a conception of a certain range of sets, we can define a set that isn't in that range.

Criticism: the view is hard to state and perhaps self-refuting (Williamson 2003).
But of course, this is hotly disputed, cf. notably Glanzberg.

NB: This problem is not peculiar to sets.
Rayo (2002) shows that the problem arises as soon as we use surrogates for plurals for which this is true:
If s is a surrogate for the Fs, and the Gs are not the Fs, then s is not a surrogate for the Gs.

<u>So a second response</u> translates the sentence in terms of sums:
$\exists y\ \forall x\ (\text{set}(x) \rightarrow (x \leq y \leftrightarrow x \notin x))$
There is a sum y which is the sum of all the sets that don't belong to themselves.

If we make no special assumptions about sets and sums, then there is no contradiction.

Now, Uzquiano (2006) shows that various assumptions concerning set theory, mereology *and* plural logic lead to paradox.
For instance, one cannot simultaneously:
- axiomatize set theory as in ZFCU, with the help of plural logic, and suppose every object is a member
- axiomatize mereology as in classical extensional mereology, with the help of plural logic, and suppose every object is a part
- suppose the parts of a set are all and only its subsets

But of course, the singularist does *not* use an extensional mereology and *denies* that plural logic is genuine, primitive.
So he won't use plural logic to axiomatize set theory and mereology.
Price to pay: set theory and mereology must then be formulated with the help of axiom schemata.
Plurals and plural logic would allow one to formulate these schemata as single axioms.
But for the singularist, this hope is *illusory*.

What are other problems for this view, if we just want to develop a truth-theory for plurals, using sums as their semantic values, while using plural sentences to talk about sets? Thoughts welcome!

Conclusion

* Radical mereological singularism:

- Because it renounces sets altogether, this makes the development of a model theory, hence of a full semantics, very difficult.

- The mereology used must not be extensional. Inspired by Link, one may use two mereologies.

- Counting problems (and similar ones) lead this form of singularism to postulate an infinite hierarchy of metalanguages.
In itself, this doesn't seem so bad, as most theories are lead, because of paradoxes, to similar hierarchies.
But once the hierarchy is in place, one wonders why not just use sets, or perhaps, sums and sets?

* Liberal mereological singularism: sums are the referents of plural expressions, but sets are employed for truth or model theory as usual.

* Set singularism: sets as referents of plural expressions

How different are they from one another?

* Two other problems for singularist approaches:
- Their ontological commitment to sums or sets.
- The threat of paradox if quantification over absolutely everything is possible.

Concerning ontological commitment, plausible stand-off: intuitions and arguments seem indecisive, one way or the other.

Concerning paradox, all depends on whether quantification over absolutely everything is possible, or whether some concepts (like set) are indefinitely extensible.

* So what about plural logic?

Plural quantification and plural predication are taken to be primitive.

Plural logic, if genuine, is attractive because of:
- its power (similar to monadic second order logic)
- its alleged ontological innocence.

But the ontological innocence of plural logic really is a matter of debate.

Also, although plural logic is powerful, its power is limited.

To get more power, should we accept 'superplural' quantifiers (Linnebo & Nicolas 2008) or even higher-order plural quantifiers? This does remain questionable.

Moreover, do the set-theoretic functions of complex types used in natural language semantics always have analogs in "higher-order" plural logic?

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